

# Condensation of charged bosons in plasma physics and cosmology

A.D.Dolgov<sup>a\*</sup>

<sup>a</sup> *University of Ferrara and INFN  
Ferrara, FE44100, Italy  
ITEP, Moscow, 117218, Russia*

## Abstract

The screening of impurities in plasma with Bose-Einstein condensate of electrically charged bosons is considered. It is shown that the screened potential is drastically different from the usual Debye one. The polarization operator of photons in plasma acquires infrared singular terms at small photon momentum and the screened potential drops down as a power of distance and even has an oscillating behavior, similar to the Friedel oscillations in plasma with degenerate fermions. The magnetic properties of the cosmological plasma with condensed W-bosons are also discussed. It is shown that W-bosons condense in the ferromagnetic state. It could lead to spontaneous magnetization of the primeval plasma. The created magnetic fields may seed galactic and intergalactic magnetic fields observed in the present-day universe.

## 1 Introduction

The potential created by an electric charge in plasma is usually described by the well known Debye formula, see e.g. refs. [1, 2]. The long range Coulomb potential transforms into the exponentially decreasing Yukawa one:

$$U(r) = \frac{Q}{4\pi r} \rightarrow \frac{Q \exp(-m_D r)}{4\pi r}, \quad (1)$$

because the time-time component of the photon propagator acquires a constant term which does not vanish when the three dimensional photon momentum goes to zero. Instead of the vacuum  $k^2$ -term the inverse propagator becomes:

$$k^2 \rightarrow k^2 + \Pi_{00}(k) = k^2 + m_D^2, \quad (2)$$

where e.g. for relativistic fermions[3]:

$$m_D^2 = e^2 \left( T^2/3 + \mu^2/\pi^2 \right). \quad (3)$$

These results are true if the fermions in plasma are not strongly degenerate and if the charged bosons do not condense. The modification of the Debye screening in the case of degenerate fermions was studied half a century ago [4], but the impact of Bose-Einstein condensate (BEC) on the screening of impurities was considered only very recently [5, 6]. Surprisingly in the presence of BEC the screened potential drastically changes and becomes even an oscillating function of distance, which decreases either exponentially or as a power of  $r$ .

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\*e-mail: dolgov@fe.infn.it

In the first part of this talk I discuss the impact of BEC of a charged scalar field on the screening of charged impurities in plasma. It is based on the works done in collaboration with A. Lepidi and G. Piccinelli [6]. The second part about ferromagnetism of the condensate of charged vector bosons is based on paper [7] of the same group. More detailed list of references can be found in these papers.

Let us consider electrically neutral plasma with large electric charge density of bosons which is compensated by charged fermions. Bosons condense when their chemical potential reaches the maximum allowed value:

$$\mu_B = m_B. \quad (4)$$

This can be easily seen from the consideration of the kinetic equation. Indeed the equilibrium distribution of bosons, if and only if their chemical potential is equal to their mass, takes the form:

$$f_B^{(eq)} = C\delta^{(3)}(\mathbf{q}) + \frac{1}{\exp[(E - m_B)/T] - 1}, \quad (5)$$

where the constant  $C$  is the amplitude of the condensate. One can check that  $f_B^{(eq)}$  annihilates the collision integral for an arbitrary  $C$ . Thus, by definition, it is an equilibrium distribution. It is worth noting that the equilibrium distributions are always functions of two parameters: temperature,  $T$ , and chemical potential,  $\mu$ , when  $\mu < m$ , and temperature  $T$ , and the amplitude of the condensate,  $C$ , when  $\mu = m$ .

We calculate the time-time component of the photon polarization operator in a simple straightforward way perturbatively solving operator equation of motion for the electromagnetic field (Maxwell equations) and averaging them over medium. One can reach the goal without applying to more refined real or imaginary time methods (for a review of these methods see e.g. Ref. [2]).

## 2 Maxwell equations in thermal bath

The standard Lagrangian of QED with interacting electromagnetic field and charged scalar and fermion fields with masses  $m_B$  and  $m_F$  respectively and with opposite electric charges  $\pm e$  has the form:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m_B^2|\phi|^2 + |(\partial_\mu + ieA_\mu)\phi|^2 + \bar{\psi}(i\cancel{\partial} - e\cancel{A} - m_F)\psi. \quad (6)$$

The equations of motion for the electromagnetic and charged scalar and spinor fields are respectively:

$$(i\cancel{\partial} - m)\psi(x) = e\cancel{A}\psi(x), \quad (7)$$

$$(\partial_\mu\partial^\mu + m^2)\phi(x) = \mathcal{J}_\phi(x), \quad (8)$$

$$\partial_\nu F^{\mu\nu}(x) = \mathcal{J}^\mu(x), \quad (9)$$

where the currents  $\mathcal{J}$  can be written as:

$$\mathcal{J}_\phi(x) = -ie\left[\partial_\mu A^\mu(x) + 2A_\mu(x)\partial^\mu\right]\phi(x) + e^2 A^\mu(x)A_\mu(x)\phi(x), \quad (10)$$

$$\begin{aligned} \mathcal{J}^\mu(x) &= -ie\left[(\phi^\dagger(x)\partial^\mu\phi(x)) - (\partial^\mu\phi^\dagger(x))\phi(x)\right] \\ &+ 2e^2 A^\mu(x)|\phi(x)|^2 - e\bar{\psi}(x)\gamma^\mu\psi(x). \end{aligned} \quad (11)$$

Here  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $\mathcal{J}^\mu$  (11) is the total electromagnetic current of bosons and fermions.

Operator equations (7) and (8) can be formally solved as:

$$\phi(x) = \phi_0(x) + \int d^4y G_B(x-y) \mathcal{J}_\phi(y), \quad (12)$$

$$\psi(x) = \psi_0(x) + \int d^4y G_F(x-y) e\mathcal{A}(y)\psi(y), \quad (13)$$

where  $G_B$  and  $G_F$  are the Green's functions of bosons and fermions respectively and the zeroth order fields satisfy the free equations of motion:

$$(\partial_\mu \partial^\mu + m_B^2)\phi_0(x) = 0, \quad (i\partial\!\!\!/ - m_F)\psi_0(x) = 0 \quad (14)$$

and are quantized in the usual way:

$$\phi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3 2E}} \left[ a(\mathbf{q})e^{-iqx} + b^\dagger(\mathbf{q})e^{iqx} \right], \quad (15)$$

$$\psi_0(x) = \int \frac{d^3q}{\sqrt{(2\pi)^3}} \sqrt{\frac{m_F}{E}} \left[ c_r(\mathbf{q})u_r(\mathbf{q})e^{-iqx} + d_r^\dagger(\mathbf{q})v_r(\mathbf{q})e^{iqx} \right]. \quad (16)$$

Substituting solutions (12,13) into eq. (9) we obtain the Maxwell equations with the lowest order corrections to the electromagnetic current:

$$\begin{aligned} \partial_\nu F^{\mu\nu}(x) &= -ie \left[ (\phi_0^\dagger(x) \partial^\mu \phi_0(x)) - (\partial^\mu \phi_0^\dagger(x)) \phi_0(x) \right] - e\bar{\psi}_0(x) \gamma^\mu \psi_0(x) \\ &- ie \phi_0^\dagger(x) \partial^\mu \left[ \int d^4y G_B(x-y) \mathcal{J}_{\phi_0}(y) \right] - ie \left[ \int d^4y G_B(x-y) \mathcal{J}_{\phi_0}(y) \right]^\dagger \partial^\mu \phi_0(x) \\ &+ ie \partial^\mu \phi_0^\dagger(x) \left[ \int d^4y G_B(x-y) \mathcal{J}_{\phi_0}(y) \right] + ie \partial^\mu \left[ \int d^4y G_B(x-y) \mathcal{J}_{\phi_0}(y) \right]^\dagger \phi_0(x) \\ &- e\bar{\psi}_0(x) \gamma^\mu \int d^4y G_F(x-y) e\mathcal{A}(y)\psi(y) - e \left[ \int d^4y \bar{\psi}_0(y) \mathcal{A}(y) G_F^*(x-y) \right] \gamma^\mu \psi_0(x) \\ &+ 2e^2 A^\mu(x) |\phi_0(x)|^2. \end{aligned} \quad (17)$$

To derive the Maxwell equations with the account of the impact of medium on the photon propagator we have to average operators  $\phi$  and  $\psi$  over the medium. The first term in eq. (17), linear in  $e$ , is non-zero if the medium is either electrically charged or possesses an electric current. We assume here that this is not the case, i.e. the medium is electrically neutral and “current-less”.

The products of creation-annihilation operators averaged over the medium have the standard form:

$$\langle a^\dagger(\mathbf{q})a(\mathbf{q}') \rangle = f_B(E_q) \delta^{(3)}(\mathbf{q} - \mathbf{q}'), \quad (18)$$

$$\langle a(\mathbf{q})a^\dagger(\mathbf{q}') \rangle = [1 + f_B(E_p)] \delta^{(3)}(\mathbf{q} - \mathbf{q}'), \quad (19)$$

$$\langle c^\dagger(\mathbf{q})c(\mathbf{q}') \rangle = f_F(E_p) \delta^{(3)}(\mathbf{q} - \mathbf{q}'), \quad (20)$$

$$\langle c(\mathbf{q})c^\dagger(\mathbf{q}') \rangle = [1 - f_F(E_p)] \delta^{(3)}(\mathbf{q} - \mathbf{q}'), \quad (21)$$

where  $f_{F,B}(E_q)$  is the energy dependent fermion/boson distribution function, which may be arbitrary since we assumed only that the medium is homogeneous and isotropic. We also assumed, as it is usually done, that the non-diagonal matrix elements of creation-annihilation operators vanish on the average due to decoherence. For the vacuum case  $f_{F,B}(E) = 0$  and we obtain the usual vacuum average values of  $\langle aa^\dagger \rangle$  and  $\langle a^\dagger a \rangle = 0$ , which from now on will be

neglected because we are interested only in the matter effects. As a result we obtain linear but non-local equation for electromagnetic field  $A_\mu(x)$ , for which it is convenient to perform the Fourier transform:

$$A^\mu(k) = \int \frac{d^4x}{(2\pi)^3} e^{-ikx} A^\mu(x). \quad (22)$$

Finally we find that field  $A^\mu(k)$  satisfies the equation

$$[k^\rho k_\rho g^{\mu\nu} - k^\mu k^\nu + \Pi^{\mu\nu}(k)] A_\nu(k) = \mathcal{J}^\mu(k), \quad (23)$$

which is equivalent to the photon equation of motion (17) in momentum space.

In this way the photon polarization tensor, which contains contributions from the charged bosons and fermions,  $\Pi_{\mu\nu}(k) = \Pi_{\mu\nu}^B(k) + \Pi_{\mu\nu}^F(k)$ , according to eq. (17), can be explicitly found in the lowest order in  $e^2$ :

$$\begin{aligned} \Pi_{\mu\nu}^B(k) = e^2 \int \frac{d^3q}{(2\pi)^3 E} [f_B(E) + \bar{f}_B(E)] & \left[ \frac{1}{2} \frac{(2q-k)_\mu (2q-k)_\nu}{(q-k)^2 - m_B^2} \right. \\ & \left. + \frac{1}{2} \frac{(2q+k)_\mu (2q+k)_\nu}{(q+k)^2 - m_B^2} - g_{\mu\nu} \right], \end{aligned} \quad (24)$$

$$\begin{aligned} \Pi_{\mu\nu}^F(k) = 2e^2 \int \frac{d^3q}{(2\pi)^3 E} [f_F(E) + \bar{f}_F(E)] & \left[ \frac{q_\nu (k+q)_\mu - q^\rho k_\rho g_{\mu\nu} + q_\mu (k+q)_\nu}{(k+q)^2 - m_F^2} \right. \\ & \left. + \frac{q_\nu (q-k)_\mu + q^\rho k_\rho g_{\mu\nu} + q_\mu (q-k)_\nu}{(k-q)^2 - m_F^2} \right]. \end{aligned} \quad (25)$$

The static properties of the medium are determined by the time-time component of the polarization tensor in the limit of  $\omega = 0$ , which can be easily calculated from the above expressions:

$$\Pi_{00}^B(k) = -\frac{e^2}{2\pi^2} \int_0^\infty \frac{dq q^2}{E} (f_B + \bar{f}_B) \left( 1 + \frac{E^2}{kq} \ln \left| \frac{2q+k}{2q-k} \right| \right), \quad (26)$$

$$\Pi_{00}^F(k) = -\frac{e^2}{\pi^2} \int_0^\infty \frac{dq q^2}{E} (f_F + \bar{f}_F) \left( 1 + \frac{E^2}{kq} \ln \left| \frac{2q+k}{2q-k} \right| \right). \quad (27)$$

Here and in what follows  $k$  and  $q$  are respectively the absolute values of the spatial component of the photon and the charged particle momenta. Expressions (26,27) coincide with the well known ones found by other methods. Our new results for screening come from an addition of the condensate term to  $f_B$ , eq. (5).

After straightforward calculations we find that the time-time component of the charge boson contribution into the photon polarization tensor at zero frequency and small  $k$  (but high  $T$ ) has the form:

$$\Pi_{00} = [k^2 + e^2(m_0^2 + m_1^3/k + m_2^4/k^2)], \quad (28)$$

where

$$\begin{aligned} m_0^2 &= 2T^2/3 + C/[(2\pi)^3 m_B] \\ m_1^3 &= m_B^2 T/2 \\ m_2^4 &= 4C m_B / (2\pi)^3. \end{aligned} \quad (29)$$

In fact the same dependence on  $k$  is true for any temperature but the coefficients  $m_j$  may be different.

### 3 Screening of electric charge

The screened potential is determined by the Fourier transformation of the photon propagator  $(k^2 - \Pi_{00})^{-1}$ :

$$U(r) = Q \int \frac{d^3k}{(2\pi)^3} \frac{\exp(i\mathbf{k}\mathbf{r})}{k^2 - \Pi_{00}(k)} = \frac{qQ}{2\pi^2} \int_0^\infty \frac{dk k^2}{k^2 - \Pi_{00}(k)} \frac{\sin kr}{kr}. \quad (30)$$

If  $\Pi_{00}$  is an even function of  $k$ , as is usually the case, the integration over  $k$  can be extended to the interval from  $-\infty$  to  $+\infty$  and the integral can be taken as a sum over residues of the integrand. For example, if the term proportional to  $1/k$  can be neglected (low temperature case),  $\Pi_{00}$  is evidently even and its poles can be easily found:

$$k_j^2 = -\frac{e^2 m_0^2}{2} \pm \sqrt{\left[ \frac{e^4 m_0^4}{4} - e^2 m_2^4 \right]} \approx \pm i e m_2^2. \quad (31)$$

The last approximate equality is formally true in the limit of vanishingly small  $e$ .

Thus in the presence of the charged Bose condensate the “Debye” poles acquire the non-zero real parts:

$$k_j = \pm \sqrt{e} m_2 \exp(\pm i\pi/4) \equiv k'_j + i k''_j, \quad (32)$$

Non-zero  $k'$  leads to the oscillating behavior of the potential

$$U(r)_j \sim Q \frac{\exp(-\sqrt{e/2} m_2 r) \cos(\sqrt{e/2} m_2 r)}{r}. \quad (33)$$

If the term proportional to  $1/k$  is present in  $\Pi_{00}$ , the calculations of the potential are slightly more complicated. Now the integration path in the complex  $k$  plane cannot be extended to  $-\infty$  but the integration should be done along the real axis from 0 to  $\infty$ , then along infinitely large quarter-circle, and along the imaginary axis from  $\infty$  to 0. The result would contain the usual contributions from the poles and other singularities (see below) in the complex  $k$ -plane and the integral over the imaginary axis. The former gives the usual exponentially decreasing potential, while the latter gives a power law decrease:

$$U \sim Q m_1^3 / (e^2 m_2^8 r^6). \quad (34)$$

Notice that the potential is inversely proportional to the electric charge squared. This is because we consider asymptotical behavior of the screened potential at large distance,  $r$ , when the parameter  $er$  formally tends to infinity.

There are some other singularities in the integrand of eq. (30), which arise from the pinching of the contour of integration over  $q$  in eq. (26) by the poles of the distribution function,  $f_B$  (5), and the branch point of the logarithm. The induced singularities in the complex  $k$ -plane give rise to the screening effects analogous to the Friedel oscillations [4]. If the first “pinch” dominates, the screened potential is:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2} \frac{e^{-z}}{\ln^2(2\sqrt{2}z)} \sin z, \quad (35)$$

where  $z = 2r\sqrt{2\pi T m_B}$ . Notice that  $U_1(r)$  is inversely proportional to  $e^2$  and formally vanishes at  $T \rightarrow 0$ , but remains finite if  $\sqrt{T m_B} r \neq 0$ .

If all pinches are comparable, the screened potential drops down as a power of distance:

$$U(r) \approx -\frac{3Q}{2e^2 T^2 m_B^3 r^6 \ln^3(\sqrt{8m_B T} r)}. \quad (36)$$

More details can be found in the second paper of Ref. [6].

## 4 Condensation of vector bosons

The condensation of the charged vector bosons [7] of the electroweak group might take place in the early universe if the cosmological lepton asymmetry was sufficiently high. Condensation of vector particles differs from that of the scalars due to the additional degree of freedom, their spin states. Depending upon the interactions between the spins, they can be either aligned or anti-aligned. These states are called respectively ferromagnetic and anti-ferromagnetic ones, see e.g. Ref. [9]. We show that  $W$ -bosons of the minimal electroweak theory condense in the ferromagnetic state and spontaneous magnetization of the primeval plasma could generate strong primordial magnetic fields on macroscopically large scales.

Recently somewhat similar problem of condensation of deuterium nuclei in astrophysics has been studied in Ref. [8]. The authors argue that the interaction between deuterium nuclei forces them into the lowest spin antiferromagnetic state.

In the minimal standard electroweak model the spin-spin interaction of  $W$ -bosons is determined by the interaction between their magnetic moments and their contact quartic coupling. The former can be found from the analogue of the Breit equation for vector particles which leads to the spin-spin potential of the form [7]:

$$U_{em}^{spin}(r) = \frac{\alpha \rho^2}{m_W^2} \left[ \frac{(\mathbf{S}_1 \cdot \mathbf{S}_2)}{r^3} - 3 \frac{(\mathbf{S}_1 \cdot \mathbf{r})(\mathbf{S}_2 \cdot \mathbf{r})}{r^5} - \frac{8\pi}{3} (\mathbf{S}_1 \cdot \mathbf{S}_2) \delta^{(3)}(\mathbf{r}) \right], \quad (37)$$

where  $\alpha = e^2/4\pi \approx 1/137$  and  $\rho$  is the ratio of the real magnetic moment of  $W$  to its value in the electroweak theory. Since the plasma is supposed to be neutral, the Coulomb interaction between the condensed  $W$  is compensated by the charged leptons.

To find the energy shift of a pair of  $W$ -bosons due to this interaction we need to average potential (37) over the  $W$ -wave function. The wave function of the condensate is supposed to be angle independent S-wave state. Thus the energy shift induced by the magnetic spin-spin interaction is expressed through the integral of potential (37) over space:

$$\delta E = \int \frac{d^3r}{V} U_{em}^{spin}(r) = -\frac{2e^2\rho^2}{3Vm_W^2} (\mathbf{S}_1 \cdot \mathbf{S}_2), \quad (38)$$

where  $V$  is the normalization volume.

Since  $S_{tot}^2 = (S_1 + S_2)^2 = 4 + 2S_1S_2$ , the average value of  $S_1S_2$  is equal to

$$S_1S_2 = S_{tot}^2/2 - 2. \quad (39)$$

For  $S_{tot} = 2$  this term is  $S_1S_2 = 1 > 0$ , while for  $S_{tot} = 0$  it is  $S_1S_2 = -2 < 0$ . Thus, if the spin-spin interaction is dominated by the interactions between the magnetic moments of  $W$  bosons, the state with their maximum total spin is energetically more favorable and  $W$ -bosons should condense in the ferromagnetic state. Such an interaction would lead to the spontaneous magnetization of the vector particles in the early universe.

Another contribution to the spin-spin interactions of  $W$  comes from their quartic self-coupling:

$$L_{4W} = \frac{e^2}{2\sin^2\theta_W} \left[ W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu - (W_\mu^\dagger W^\mu)^2 \right] = \frac{e^2 (\mathbf{W}^\dagger \times \mathbf{W})^2}{2\sin^2\theta_W}. \quad (40)$$

It is assumed here that  $\partial_\mu W^\mu = 0$  and thus only the spatial 3-vector  $\mathbf{W}$  is non-vanishing, while  $W_t = 0$ . The Fourier transform of this term with proper (nonrelativistic) normalization leads to

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W} (\mathbf{S}_1 \mathbf{S}_2) \delta^{(3)}(\mathbf{r}). \quad (41)$$

Thus the quartic self-coupling of  $W$  contributes only to the spin-spin interaction whose sign is antiferromagnetic.

In the minimal standard model the interaction between the magnetic moments of  $W$  dominates and the condensed  $W$ -bosons have ferromagnetic behavior. However, in some modification of the standard model antiferromagnetic behavior is possible and  $W$  would condense in the state with zero or microscopically small total spin. In this case classical vector field of  $W$ -bosons could not be created, though they would still make the Bose condensed state.

The exchange of  $Z^0$ -bosons may also contribute to spin-spin interactions of  $W$ . It can be shown that for non-relativistic  $Z$  this contribution vanishes [7]. However, if the momentum carried by the virtual  $Z$  is non-negligible in comparison with its mass, the contributions of  $Z$  and photon exchanges are similar.

The long range interactions between the magnetic moments of  $W$ -bosons, in principle, can be screened by the plasma effects. However, in contrast to electric interactions, which are Debye-like screened, magnetic interactions in pure electrodynamics (or in any other Abelian theory) are known to remain unscreened. On the other hand, in non-Abelian theories the screening may occur in higher orders of perturbation theory due to the violent infrared singularities, which make impossible perturbative calculations [10]. The screening may potentially change the relative strength of the electromagnetic spin-spin coupling, which is affected by screening effects, with respect to the local quartic,  $W^4$ -coupling which is not screened. However, in the broken phase the system is reduced to the usual electrodynamics, where screening is absent and  $W$ -bosons would condense in the ferromagnetic state. In the unbroken phase of the electroweak theory the answer is not yet known.

If the magnetic interaction between the spins of  $W$ -bosons is screened the potential describing the magnetic spin-spin interaction is related to amplitude (37) with a modified photon propagator. So it can be written as:

$$U_{em}^{(spin)}(\mathbf{r}) = -\frac{e^2 \rho^2}{m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{\exp(i\mathbf{q}\mathbf{r})}{(q^2 + \Pi_{ss}(\mathbf{q}))} \left[ q^2 (\mathbf{S}_1 \cdot \mathbf{S}_2) - (\mathbf{S}_1 \cdot \mathbf{q})(\mathbf{S}_2 \cdot \mathbf{q}) \right], \quad (42)$$

where  $\Pi_{ss}$  is the plasma correction to the space-space component of the photon propagator.

If, as above, we assume that the wave function of  $W$ -bosons is space independent and average this potential over space, we obtain the following expression for the spin-spin part of the energy shift:

$$\delta E = \int \frac{d^3 r}{V} U_{em}^{(spin)}(\mathbf{r}) = -\frac{e^2 \rho^2}{V m_W^2} \int \frac{d^3 q}{(2\pi)^3} \delta^{(3)}(\mathbf{q}) \frac{q^2 (\mathbf{S}_1 \cdot \mathbf{S}_2) - (\mathbf{q} \cdot \mathbf{S}_1)(\mathbf{q} \cdot \mathbf{S}_2)}{q^2 + \Pi_{ss}(\mathbf{q})} \quad (43)$$

Clearly  $\delta E$  vanishes if  $\Pi_{ss}$  is non-zero at  $q = 0$ . Of course, this is an unphysical conclusion, because the integration over  $r$  should be done with some finite upper limit,  $r_{max} = l$ , presumably equal to the average distance between the  $W$  bosons. So instead of the delta-function,  $\delta^{(3)}(\mathbf{q})$ , we obtain:

$$\int_0^l d^3 r \exp(i\mathbf{q}\mathbf{r}) = \frac{4\pi}{q^3} [\sin(ql) - ql \cos(ql)]. \quad (44)$$

The energy shift is given by the expression:

$$\delta E = -4\pi \frac{e^2 \rho^2}{V m_W^2} S_1^i S_2^j \int \frac{d^3 q}{(2\pi)^3} \frac{[\sin(ql) - ql \cos(ql)] [q^2 \delta_{ij} - q^i q^j]}{q^3 [q^2 + \Pi_{ss}(q)]}, \quad (45)$$

where  $V = 4\pi l^3/3$ .

When we average over an angle independent wave function, e.g. S-wave for the condensate, the non-vanishing part of the integral in Eq. (45) is proportional to the Kronecker delta, hence:

$$\delta E = S_1^i S_2^j A \delta^{ij}, \quad (46)$$

where the coefficient  $A$  can be calculated by taking trace of Eq. (45):

$$Tr(A\delta^{ij}) = 3A = -8\pi \frac{e^2 \rho^2}{m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{[q \sin(ql) - q^2 l \cos(ql)]}{q^2 [q^2 + \Pi_{ss}(q)]}. \quad (47)$$

Hence the energy shift of a pair of  $W$ -bosons in S-wave state due to the spin-spin interaction is:

$$\delta E = -(\mathbf{S}_1 \cdot \mathbf{S}_2) \frac{8\pi e^2 \rho^2}{3V m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{[\sin(ql) - ql \cos(ql)]}{q [q^2 + \Pi_{ss}(q)]}. \quad (48)$$

Introducing the new integration variable  $x = ql$ , we can rewrite it as:

$$\delta E = -(\mathbf{S}_1 \cdot \mathbf{S}_2) \frac{4e^2 \rho^2}{3\pi V m_W^2} \int_0^\infty \frac{dx}{x^2 + l^2 \Pi_{ss}(x/l)} [x \sin x + l^2 \Pi_{ss}(x/l) \cos x], \quad (49)$$

We used here the usual regularization of divergent integrals:  $\exp(\pm iql) \rightarrow \exp(\pm iql - \epsilon q)$  with  $\epsilon \rightarrow 0$ . With such regularization  $\int_0^\infty dx \cos(x) = 0$ .

Evidently, if  $\Pi_{ss} = 0$ , we obtain the same result as that found above. In fact the necessary condition for obtaining the “unscreened” result is  $l^2 \Pi_{ss}(x/l) \ll 1$ , but for a large  $l^2 \Pi_{ss}$  the electromagnetic part of the spin-spin interaction can be suppressed enough to change the ferromagnetic behavior into the antiferromagnetic one. This might take place at high temperatures above the EW phase transition when the Higgs condensate is destroyed and the masses of  $W$  and  $Z$  appear only as a result of temperature and density corrections and thus are relatively small. The quantitative statement depends upon the modification of the space-space part of the photon propagator in presence of the Bose condensate of charged  $W$ . As far as we know, this modification has not yet been found.

If  $W$ -bosons make a ferromagnetic state, the primeval plasma, where such bosons condensed (possibly due to a large cosmological lepton asymmetry), could be spontaneously magnetized, as it happens in usual ferromagnets. The typical size of the magnetic domains is determined by the cosmological horizon at the moment of the condensate evaporation. The latter takes place when the neutrino chemical potential, which scales as temperature in the course of cosmological cooling down, becomes smaller than the  $W$  mass at this temperature.

The large scale magnetic field, produced by the ferromagnetism of  $W$ -bosons, might survive after the decay of the condensate due to the conservation of the magnetic flux in the primeval plasma because of its high electric conductivity. Such magnetic fields, which were uniform at macroscopically large scales at formation, may be the seeds for the observed galactic or intergalactic magnetic fields at astronomically large scales. Evidently the characteristic size of magnetic domains at production is much smaller than the galactic size, even with an account of the cosmological stretching out. Nevertheless, magnetic fields homogeneous at astronomical scales may be created by chaotic reconnection (Brownian motion) of the magnetic field lines at much smaller scales but by an expense of the field amplitude. Such a mechanism could be a competing alternative among many other attempts to solve the mystery of the generation of large scale magnetic fields in cosmology, for a review see ref. [11].

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